

A LOTKA-VOLTERRA MODEL OF THE EVOLUTIONARY DYNAMICS OF COMPOSITIONALITY MARKERS

ANDREAS BAUMANN, CHRISTINA PRÖMER

*Department of English, University of Vienna,
Vienna, Austria*

andreas.baumann@univie.ac.at, christina.proemer@univie.ac.at,

KAMIL KAŻMIERSKI

*Faculty of English, Adam Mickiewicz University,
Poznań, Poland*

kamil.kazmierski@wa.amu.edu.pl

NIKOLAUS RITT

*Department of English, University of Vienna,
Vienna, Austria*

nikolaus.ritt@univie.ac.at

Morpho-syntactic boundaries can either be signaled by alignment to boundaries in regular prosodic patterns or by being ‘irregularly’ misaligned, in which case they are often signaled instead through highly dispreferred, or marked, structures such as consonant clusters. In some languages these structures additionally appear in simple forms, which compromises their compositionality-signaling function. This paper models the dynamics of such structures in complex and simple forms by means of a Lotka-Volterra model, which is analyzed evolutionarily. Finally, the evolutionary dynamics of the model are tested against diachronic language data.

1. Introduction

Syntactic and morpho-semantic compositionality is often mirrored, and thereby signaled, prosodically: for example, morpho-syntactic constituents may correspond to rhythmic feet (i.e. alternating sequences of stressed and unstressed syllables), or to regular sequences of (typically CV-) syllables (Schlüter 2005). In a number of languages, however, compositionality is signaled by irregular and phonologically marked structures, such as consonant clusters spanning

inflectional or derivational morpheme boundaries (English /md/, as in *seem-ed*, for instance). Their signaling function thus compensates for the comparably high costs involved in their production and perception. Nevertheless, such structures can also appear in simple forms (e.g. English /nd/ both in *plann-ed* and in *hand*). Since this obviously compromises their signaling function, the prediction has been made that clusters which span (and signal) morpheme boundaries select against morpheme internal homophones, and vice versa (Dressler & Dziubalska-Kolaczyk 2006, Calderone et al. 2014). On the other hand, however, it has been argued that consonant clusters in simple and complex forms benefit from each other through analogy effects (Hogg & McCully 1987).

Conceptualizing structures acting as compositionality markers as linguistic replicators (Croft 2000, Kirby 2013), the present paper introduces a mathematical model of the dynamics in the cultural evolution of these structures, which takes both of the cognitive mechanisms described above into account. It models both short-term dynamics on the ecological time scale – i.e. the speech-based interaction of potential compositionality markers in complex and simple forms – as well as long-term dynamics on the evolutionary time scale – i.e. the change in the distribution of potential compositionality markers in complex and simple forms. The model is then tested against historical language data.

2. Formulation and analysis of the model

2.1. Ecological dynamics

The ecological short-term dynamics of potential compositionality markers are modeled in terms of a two-dimensional Lotka-Volterra dynamical system in continuous time. Lotka-Volterra models are simple and have been fruitfully applied in linguistics and cultural evolution (Cavalli-Sforza & Feldman 1981, Nowak et al. 2001, Solé et al. 2010, Zhang & Gong 2013). While of limited use for precise quantitative predictions, Lotka-Volterra equations are highly valuable for qualitative analyses. We distinguish between items that actually are signaling complexity (e.g. instances of the consonant cluster type /nd/ spanning a morpheme boundary), and items with the very same structure, that do not (e.g. instances of /nd/ within morphemes). The dynamics of the size of the population of morphologically complex tokens C and the size of the population of tokens of their morphologically simple counterparts S is given by

$$\begin{pmatrix} \dot{C} \\ \dot{S} \end{pmatrix} = A(\lambda) \cdot \begin{pmatrix} C \\ S \end{pmatrix},$$

where the matrix

$$A(\lambda) = \begin{pmatrix} r_c (1 - \lambda) \left(1 - \frac{c}{K}\right) - d & a \\ a & r_s \lambda \left(1 - \frac{s}{K}\right) - d \end{pmatrix}$$

is a function of λ , parametrizing the amount of simple items the structure occurs in (see below). Consider the dynamics of S first. The per token growth rate \dot{S}/S measures the number of tokens in simple items produced or lost per time unit. It depends on three components: (i) production due to replication (uttering and recognizing tokens), (ii) production due to analogy, and (iii) disappearance. The intrinsic reproduction rate $r_s > 0$ measures the number of offspring tokens in simple items produced per token per time unit. It depends on (a) the average frequency of each item type the structure occurs in, (b) the probability of uttering the structure correctly if it occurs in a simple item, (c) the probability of recognizing and processing the uttered structure, and (d) the number of simple items in which the structure occurs, i.e. the number of structure hosts. Let $\lambda \in [0,1]$ be the fraction of actual structure host items. It can increase when potential hosts become actual ones, e.g. through vowel deletion (/nød/ → /nd/), and decrease when hosts for instance undergo vowel insertion (/nd/ → /nød/). Finally, the production of such structures in simple items is bounded by a maximal number of possible tokens K , reflecting the total number of potential occurrences in speech utterances (which is clearly limited, not least due to physical reasons). It is assumed that the reproduction rate in the number of simple host structures decreases linearly in K .

In addition to reproduction, tokens in simple items can be produced via analogy to tokens in compositional items at a rate of $a > 0$ (Hogg & McCully 1987). Finally, structures can disappear at a rate of $d > 0$ tokens per time unit. This may happen (a) due to the death of a speaker or (b) when a memorized token is forgotten. Since speakers die, the disappearance rate must be strictly positive, even if speakers never forget tokens.

For complex items, the parameters $r_c, d, a, K > 0$ are defined – *mutatis mutandis* – just as for simple ones. It is assumed that the type frequency of simple items, represented by the fraction of actual structure hosts λ , affects the successful processing (i.e. the decomposition) of complex ones: if the same structure occurs in a large number of simple types, it will be more difficult to recognize the compositionality of the complex items in which it occurs (Dressler & Dziubalska-Kołaczyk 2006). This affects their reproduction rate. For the sake of simplicity we assume it to decrease linearly in λ .

Since $a > 0$, the system above describes a mutually supporting relationship between the two subpopulations. Hence, there exist four population-dynamical equilibria, one of which is an internal sink $(\hat{C}, \hat{S}) > 0$, i.e. both populations stably coexist, as long as a and d are sufficiently small (Hofbauer & Sigmund 1998).

2.2. Evolutionary dynamics

Since the number of actual structure hosts may change diachronically, the model in the previous section above does not suffice for investigating the evolutionary long-term dynamics of potential compositionality markers. Suppose the fraction of actual hosts λ changes to a new value λ' due to replication errors, such as vowel insertion or deletion in the case of consonant clusters. In this way, the configurations of replicating structures and their hosts defined by λ and λ' , respectively, can be conceptualized as two different strategies which define pairs of subpopulations (C_λ, S_λ) and $(C_{\lambda'}, S_{\lambda'})$.

The question is now, whether the new mutant strategy λ' successfully spreads and establishes itself in the language, and if so, what the evolutionary trajectory of this proportion looks like in the long run.

In order to tackle this question, an evolutionary invasion analysis is conducted on the Lotka-Volterra system above (Geritz et al. 1998, Dercole & Rinaldi 2008). For each pair of strategies (λ, λ') the so-called invasion fitness $r(\lambda, \lambda')$ is defined as the expected growth-rate of an initially rare mutant population characterized by a strategy λ' exposed to an environment determined by a resident population characterized by strategy λ at its population dynamical attractor $(\hat{C}_\lambda, \hat{S}_\lambda)$. In the case of the present dynamical system it is sufficient to derive the sign equivalent and algebraically simple expression

$$s(\lambda, \lambda') := \det(A(\lambda))|_{(C_\lambda, S_\lambda) = (\hat{C}_\lambda, \hat{S}_\lambda)},$$

(see Hoyle & Bowers 2008: Appendix B). It can be shown that if the difference between two successive strategies is sufficiently small, then $s(\lambda, \lambda') > 0$ implies that the mutant population successfully invades and replaces the resident population and the mutant population converges to its population-dynamical attractor $(\hat{C}_{\lambda'}, \hat{S}_{\lambda'})$, becoming the new resident population to be invaded by successive mutant populations (Geritz et al. 1998). If $s(\lambda, \lambda') \leq 0$ then nothing changes and the strategy stays the same. The evolutionary dynamics of strategy λ are thus fully determined by $s(\lambda, \lambda')$ and evolutionary equilibria are those points in which

$$\partial s(\lambda, \lambda') / \partial \lambda' |_{\lambda' = \lambda} = 0,$$

i.e. the fitness landscape of λ' determined by the resident λ is locally flat. A search for evolutionary equilibria and a subsequent stability analysis reveals that there exists an evolutionarily stable and attracting equilibrium at

$$\lambda_{int} = \frac{1}{2}$$

and two evolutionary repellors close to the boundaries. Hence the strategy evolves either towards the internal state λ_{int} or towards the boundaries $\lambda_0 = 0$ or $\lambda_1 = 1$. For a given λ one can easily compute the fraction of structures in simple items

$$f(\lambda) = \frac{\hat{S}_\lambda}{\hat{S}_\lambda + \hat{C}_\lambda},$$

shown in Figure 1a. Given the population dynamical equilibria for each λ it can be derived that $0 < f(\lambda_{int}) < 1$, $f(\lambda_0) = 0$ and $f(\lambda_1) = 1$, which means that – depending on their respective starting points – structures will evolve such that they (a) mark complexity unambiguously, (b) mark complexity ambiguously, or (c) never mark complexity (Fig. 1). While possibilities (a) and (c) are what is expected in the literature (Dressler et al. 2010), the possibility of diachronically stable ambiguous complexity markers is less expected. In the model, this is a reflex of the mutual relationship between the two subpopulations.

The fraction f can be estimated from diachronic language data and compared to the evolutionary trajectory of $f(\lambda)$ which is predicted from the evolutionary invasion analysis of the model. This is what will be done in the subsequent section.

3. Evaluating the model: the case of word-final consonant clusters

3.1. Data description

The model is tested against historical language data drawn from a diachronically layered data set of Middle English and Early Modern English words ending in consonant clusters. As already outlined in the introduction, English final consonant clusters are good examples of (potential) compositionality markers. In some cases they unambiguously signal morphological complexity (e.g. /md/ as in *seem-ed*, which never occurs in simple forms), while in other cases they never signal complexity (e.g. /mp/ as in *lamp*). In addition, there are word-final consonant clusters that can occur both in complex as well as in simple forms (e.g. /nd/ as in *plann-ed* and *hand*).

The Middle English period serves as a suitable testing ground for the present model, since due to the process of schwa loss, which started in the 12th century, a huge number of word-final consonant clusters were newly created. Due to this

abrupt change in the language a number of speech-driven repair processes are to be expected. Hence, if the interplay of compositionality marking sequences and those appearing in simple items indeed depends on the cognitive forces outlined in the introduction and incorporated into the model, then the predictions from the model and the development of the historical English data should coincide.

The words containing word-final consonant clusters were extracted from the Penn Helsinki Corpora of Middle English (Kroch & Taylor 2000) and Early Modern English (Kroch et al. 2004) and manually labeled as either complex (i.e. signaling compositionality) or simple (i.e. occurring within lexical items) (see Dressler et al. 2010). In total, approximately 240.000 tokens were included in the data set. The data set was then divided into 6 discrete subsets, one for each century from the 12th to the 17th century.

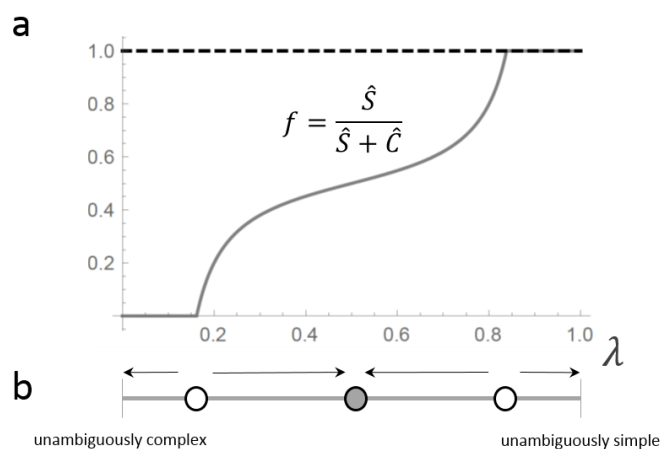


Figure 1. (a) The proportion of the token frequency of replicators in simple items among replicators appearing in simple and complex forms depending on the evolving parameter λ ($r_c = r_s = 10, a = .01, d = 2, K = 50$). (b) Representation of the evolutionary dynamics of λ .

3.2. Historical trajectories and invasion-analysis simulations

For each century and for each consonant cluster type (e.g. /md/, /mp/, and /nd/) the fraction $f = S/(S + C)$ was estimated in order to compute the respective historical trajectories (Fig. 2a). After that the frequency distributions of all f -values were determined for each century (upper histograms in Fig. 2c).

The initial values of the simulated trajectories were drawn from the empirical distribution of the 12th century data. The trajectories were then simulated according to the invasion-analysis procedure outlined in 2.2. As expected, the trajectories approach the boundaries as well as a medial strategy (Fig. 2b). Figure

2c illustrates that the final distribution of the simulated f -values is similar to the distribution of the 17th century data (small to medium difference at $V^2 = .18, \chi^2 = 16.86, N = 160$, after 15 simulation steps). For pairwise *post-hoc* comparisons of the distributions, f -values were grouped together into 5 equally sized bins. This is motivated by the classification of complexity markers proposed by Dressler et al. (2010), in which consonant clusters are either (i) reliably signaling complexity, (ii) signaling complexity by weak default, (iii) ambiguous, (iv) simple by weak default, or (v) almost always simple.

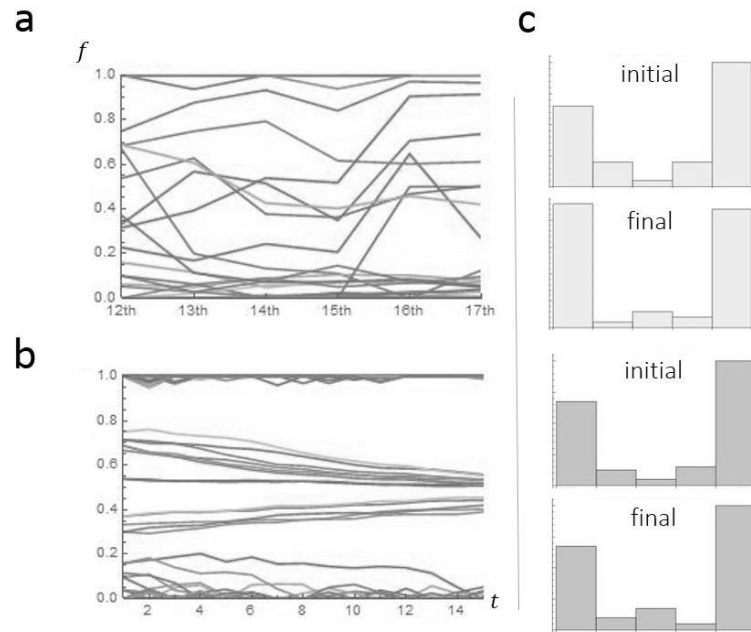


Figure 2. (a) Historical trajectories of the fractions f of $N_{hist} = 42$ to 93 different word-final consonant cluster types (period-wise sample sizes N_{hist} depending on the available data). (b) Invasion-analysis simulation of the evolution of $f(\lambda)$ of $N_{sim} = 80$ independent sequence types for $t = 15$ simulation steps ($r_c = r_s = 10, a = .01, d = 2, K = 50$; population dynamical parameters were chosen *ad hoc*; random drift according to a uniform distribution $\sim U(\pm .01)$ was included to allow for more realistic random fluctuations). Initial values are drawn from the initial historical distribution. (c) Initial and final distributions of the historical (light gray) and simulated (dark gray) f -values, respectively. The difference between the empirical distributions ($V^2 = .18, \chi^2 = 16.86$) is larger than the difference between the final ones ($V^2 = .10, \chi^2 = 15.33$).

The majority of cluster types is located at the boundaries, thus either unambiguously marking complexity or unambiguously appearing in simple forms only. However, as expected, a number of consonant clusters, namely those starting at intermediate f -values, approach the medial ambiguous strategy.

Note that in contrast to the simulated trajectories, the historical English data exhibit properties which contradict the predictions from the model, such as cluster specific trajectories that cross the threshold in the middle of the $[0,1]$ range of f . This can be attributed to the fact that single linguistic replicators, such as certain consonant-cluster types, are also influenced by factors other than analogy and complexity signaling, as for instance major changes in the inflectional system. Although it cannot be guaranteed, we hope that by studying a diverse and comparably large sample of different cluster types, such effects are largely ruled out when looking at the English consonant-cluster inventory as a whole.

4. Conclusion

In this paper a model of the dynamics of linguistic structures that potentially signal compositionality and assist the speaker in the decomposition of larger units into subunits has been proposed. Two counteracting cognitive forces, namely inhibiting semiotic ambiguity effects and analogy, have been incorporated into the model. An analysis of the model as well as simulations reveal that there are three possible stable strategies: potential compositionality-signaling sequences (a) solely appear in complex forms, (b) appear in a fair share of complex forms, or (c) never appear in complex forms. This result is supported by diachronic English language data. In particular, our findings suggest that the coexistence of certain ambiguous configurations – which are *prima facie* disadvantaged in terms of language production and perception – and structurally similar non-ambiguous configurations in language does not necessarily represent an accidental inconsistency, but can rather be explained as a natural reflex of the interaction of well-known cognitive mechanisms.

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