

THE EMERGENCE OF THE PROGRESSIVE TO IMPERFECTIVE DIACHRONIC CYCLE IN REINFORCEMENT-LEARNING AGENTS

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Deo (2015) offers a model within the framework of evolutionary game theory for the analysis of an attested phenomenon in semantic change: the progressive to imperfective cycle of shifts. While Deo studies the evolutionary dynamics of four preselected types of progressive-imperfective grammars, we investigate which types of grammars would emerge from the first principles in a population of agents under reinforcement learning. In our model, the actual progressive-to-imperfective cycle arises from such atomic interactions between learner agents after the addition of several simple assumptions to the basic game-theoretic model. The most important such addition concerns the problem of why the progressive but never the habitual generalizes to the broad imperfective. Deo (2015) conjectured that this might be due to children being more frequently exposed to progressive-type contexts than habitual-type ones. Our model vindicates Deo's conjecture: early asymmetrical exposure derives the asymmetry between the progressive and the habitual, wherein only the former gives rise to a diachronic cycle.

1. Introduction

It is a well-known typological observation that languages without a distinct progressive (PROG) morphology realize the communicative function of the PROG through the imperfective (IMP) aspect (if morphologically instantiated). This primarily motivates treating the PROG as a subdomain of the IMP (cf. Comrie, 1976). In Russian, the imperfective form licenses a PROG interpretation, while the same form refers to a habitual/generic (HAB/GEN) situation. In languages which have both the PROG and the IMP aspects, the IMP often does not license a PROG reading, such as in English. However, in languages with a less grammaticalized PROG marker, as in German, Dutch, or Shakespearean English, IMP still allows PROG interpretations. A crosslinguistically robust generalization is as follows: functional elements restricted to PROG reading semantically generalize to license IMP readings such as the HAB/GEN or the stative. This generalization has been attested according to data from, e.g., Turkish (Göksel & Kerslake, 2005,

- (1) a. saat ikide çalış-iyor-du-m
At two o'clock work-PROG-PST.COP-1.SG
'A two o'clock I was working.'
- b. genekkikle iki saat çalış-ir-di-m
Usually for two hours work-IMP-PST.COP-1.SG
'I would usually work for two hours.'
- (2) sen Ömer'i benden daha iyi tan-iyor-du-n
you Omer me better than know-PROG-PST.COP-2.SG
'You knew (lit. were knowing) Ömer better than me.'

p. 331), as shown in (1) and (2). The verb form with PROG *-(I)yor* in (1a) refers to an ongoing eventuality, while the inflected verb with IMP *-(I)r* in (1b) refers to a HAB reading. Recently, the PROG *-(I)yor* has begun to license a wider range of readings, notably in everyday language. (2) shows that *-(I)yor* occurs with a stative verb 'know'. Such data indicate that the Turkish PROG is expanding to semantically overlap with the domain of the IMP Aorist *-(I)r*, thus instantiating the PROG-to-IMP shift (Bybee, Perkins, & Pagliuca, 1994).

2. Progressive-to-Imperfective Cycle

Such typological data motivate positing a cyclic diachronic process (Table 1). This cycle starts with the language having only one broad imperfective form covering all imperfective meanings, (a). Then an optional progressive form is innovated, (b); it becomes obligatory for progressive meanings, (c); and at the last stage, (d), it generalizes and takes the semantic place of the old broad-imperfective form. Note that (a) and (d) are identical except for the formal exponents of IMP.

The four states (a-d) can be intuitively regarded as distinct *strategies* for communicating *phenomenal* (facts of local import, pertaining to specific times) and *structural* (stable facts that characterize the world as a whole) sub-meanings Goldsmith and Woisetschlaeger (1982) within the imperfective domain. In systems with two forms, namely emergent-PROG and categorical-PROG, the choice of form helps the hearer to correctly identify the speaker's intended sub-meaning. The zero-PROG and generalized-PROG strategies use a single form while relying on the hearer's understanding of contextual cues for successful communication.

Importantly, PROG induces a cycle through (a-d), but habitual HAB, though also being more specific than the broad imperfective IMP, does not eventually generalize to IMP, Deo (2015). In other words, there is no (d)-type stage for HAB, and therefore no HAB-to-IMP cycle.

Deo (2015) sets up an evolutionary game-theoretic model for studying the dynamics of these systems. She defines a simple game representing individual interactions of agents with different grammars, in the tradition of game-theoretic models of communication (cf. Benz, Jäger, and van Rooij (2006) for an overview.) Deo then lifts this game to an evolutionary game that represents the dynamics of

Table 1. The PROG-to-IMP historical cycle and sample languages.

	form(s)	strategy type	sample languages
(a)	X_{imp}	zero-PROG	Russian, Arabic
(b)	$(Y_{prog})X_{imp}$	emergent-PROG	German, Dutch
(c)	Y_{prog}, X_{imp}	categorical-PROG	English, Swahili
(d)	Y_{imp}	generalized-PROG	Turkish, Tigre

grammar retention and change over long temporal horizons. The goal of this is to model the cyclic semantic shift as in Table 1.

Both the basic and the evolutionary games are defined by Deo only for four strategies (i.e. grammars) corresponding to (a-d) above, without considering any other possibilities. In this paper, we investigate systems with PROG, IMP and HAB at a more granular level, without hardcoding the desired strategies. We consider populations of agents that build their own grammars through reinforcement learning on the basis of speaker-hearer interactions with other agents in the model. In particular, we investigate (i) which strategies actually arise, and what assumptions would rule out the ones that are not empirically observed in human languages, as well as (ii) how to induce the PROG-to-IMP, but crucially not the HAB-to-IMP cycle. Thus, we effectively provide microfoundations for Deo’s macro model of the progressive-imperfective cycle.

3. Deo’s Model

Deo’s original model consists of two parts. First, the *Basic Imperfective Game* defines communicative success in interactions between speakers of different IMP-PROG grammars. Second, the *Evolutionary Imperfective Game* models what happens in large populations of speakers who communicate according to the model of the Basic Imperfective Game, using a replicator-mutator rule for the evolution of aggregate shares of selected grammars in the population. We instead plug the Basic Imperfective Game into a reinforcement-learning framework, deriving evolutionary behavior directly from atomic interactions. The rest of this section describes the setup of the Basic Imperfective Game, without the restriction to particular strategies. For more details on the behavior of the Evolutionary Imperfective Game, we refer the reader to Deo (2015) and Yanovich (2015).

3.1. The Basic Imperfective Game

The *Basic Imperfective Game* is modeled as a *signaling game*^a Lewis (1969), a game-theoretic model that depicts the communication situation between a speaker S and a hearer H . The Basic Imperfective Game can be given as $BIG = \langle (\mathbf{S}, \mathbf{H}), C, T, F, P, U_S, U_H \rangle$. $T = \{t_s, t_p\}$ a set of two states to be signalled (*s(tructural)* and *p(henomenal)*). Signals are $F = \{f_{pr}, f_{im}\}$ (note that indices,

^aThe signaling game model proved itself useful for the analysis of phenomena in language change (cf. Ahern, 2014; Jäger, 2007, 2008; Quinley & Mühlenernd, 2012).

Table 2. All possible 16 speaker (left) and 16 hearer (right) strategies. Of those, Deo (2015) considers only 4 speaker and 3 hearer strategies (highlighted in gray).

	C_p	C_p	C_s	C_s		C_p	C_p	C_s	C_s
	t_p	t_s	t_p	t_s		f_{pr}	f_{im}	f_{pr}	f_{im}
S_0	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{im}</i>	H_0	t_p	t_p	t_p	t_p
S_1	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	H_1	t_p	t_p	t_p	t_s
S_2	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	H_2	t_p	t_p	t_s	t_p
S_3	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	H_3	t_p	t_p	t_s	t_s
S_4	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{im}</i>	H_4	t_p	t_s	t_p	t_p
S_5	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	H_5	t_p	t_s	t_p	t_s
S_6	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	H_6	t_p	t_s	t_s	t_p
S_7	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	H_7	t_p	t_s	t_s	t_s
S_8	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{im}</i>	H_8	t_s	t_p	t_p	t_p
S_9	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	H_9	t_s	t_p	t_p	t_s
S_{10}	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	H_{10}	t_s	t_p	t_s	t_p
S_{11}	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	H_{11}	t_s	t_p	t_s	t_s
S_{12}	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{im}</i>	H_{12}	t_s	t_s	t_p	t_p
S_{13}	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	<i>f_{pr}</i>	H_{13}	t_s	t_s	t_p	t_s
S_{14}	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{im}</i>	H_{14}	t_s	t_s	t_s	t_p
S_{15}	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	<i>f_{pr}</i>	H_{15}	t_s	t_s	t_s	t_s

while suggestive, by themselves do not have a meaning). Contexts $C = \{c_s, c_p\}$ model the fact that realistically, certain communication contexts highly favor phenomenal or structural meanings. Deo sets to $P(t_s|c_s) = P(t_p|c_p) = .9$. Speaker strategies $S_{\in \mathbf{S}} : T \times C \rightarrow F$, and hearer strategies $H_{\in \mathbf{H}} : F \times C \rightarrow T$ define mappings from states to signals, and from signals to states respectively. On both the speaker and hearer side, the strategy may take into account the current context drawn from C , hence it is also an argument. Finally, $U_S, U_H : T \times \mathbf{S} \times \mathbf{H} \rightarrow \mathbb{R}$ are the utility functions for the speaker and hearer. The hearer's function U_H is the δ_t -function of Jäger (2007) that returns 1 if the hearer correctly guessed the state that the speaker signalled (communicative success), and 0 otherwise (failure). The speaker function U_S is similar, but also includes a penalty of k always applying to a speaker who uses a two-form system as opposed to a single-form one. (In our implementation in the learning setting, we drop cost k from the basic model, but manipulate it in more complex models of Experiments III and IV.)

Speaker strategies define which of the two signals f_{im}, f_{pr} is to be used for every one of the four pairs (t, c) . Thus there are theoretically $2^4 = 16$ different speaker strategies. Similarly, there are 16 hearer strategies that define how to guess state t given a pair of signal f and context c . All possible S and H are shown in Table 2. We highlight the 4 speaker and 3 hearer strategies to which Deo (2015) restricts attention in her modeling. These strategies compose the ‘progressive \gg imperfective cycling path’ (PROG-path) that passes through the speaker strategies $S_0 \rightarrow S_2 \rightarrow S_{10} \rightarrow S_{15}$ and the hearer strategies $H_3 \rightarrow H_1 \rightarrow H_5 \rightarrow H_3$.

In contrast, we will use the full strategy space, and show that there exist conditions under which exactly the PROG-to-IMP diachronic cycle emerges. Pairs

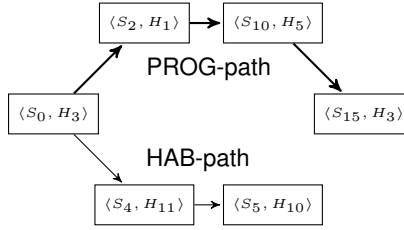


Figure 1. The ‘progressive \gg imperfective’ cycling path (*PROG-path*, thick arrows) and the alternative deadlock path (*HAB-path*, thin arrows).

of strategies which will also play an important role in our modeling also include linguistically unattested $\langle S_6, H_9 \rangle$ (“surprisal” strategy for signalling that the intended state is disfavored by the context), as well as HAB-related pairs $\langle S_4, H_{11} \rangle$ (emergent-HAB) and $\langle S_5, H_{10} \rangle$ (categorical-HAB). The question for us is: under which conditions do the trajectories of change in Fig. 1 emerge, with the cycling *PROG-path* and deadlocking *HAB-path*, and no other trajectories do?

4. Strategy Selection: a Game-Theoretic Analysis

In our simulation experiments we consider a population of 20 agents which communicate with each other via the Basic Imperfective Game over the full strategy space in Table 2. Agents have a learning memory and update their behavior via reinforcement learning (c.f. Roth & Erev, 1995)^b. Agents have a maximal age A_{max} which defines the number of rounds of play after which they are replaced by a new agent with an empty memory. We set $A_{max} = 5,000$. To have a heterogeneous ‘age structure’, we initialize the model by randomly assigning ages between 0 and 5,000 to each agent. At the beginning of the simulation, the agent have only one form f_{im} at their disposal. After 1,000 simulation steps^c the second form f_{pr} is introduced. Below, we described 4 lines of experiments we conducted, each subsequent one building more assumptions into the model. Each line consisted of 100 runs under identical conditions.

^bThe reinforcement learning model is implemented as an urn model. Each agent has (i) 4 speaker urns for each context-state combination, and (ii) 4 hearer urns for each context-form combination. Urns contain balls of two types corresponding to two signals. Those encode information about past successes, namely cumulative reward. When agents play a game with each other, they make a probabilistic choice (of form or of guessed state) in dependence of the appropriate urn’s current contents, and afterwards update their urns in dependence of the communicative success. Note that in this model agents (i) play probabilistic strategies, and (ii) do not learn pure strategies as such, but approximate them in the long run. The distance of a probabilistic to a pure strategy can be measured, e.g. by the Hellinger distance (Hellinger, 1909). For ease of exposition, we say that an agent ‘uses’ a particular pure strategy if it is the Hellinger-closest to her current probabilistic strategy.

^cFor each simulation step each agent chosen as a speaker plays the Basic Imperfective Game with a randomly chosen hearer. Context c_i is chosen randomly, and then state t is drawn randomly according to the probability distribution $P(t|c_i)$. After each interaction the agents’ urns are updated.

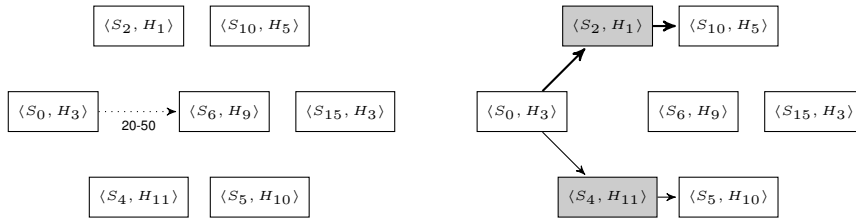


Figure 2. **Left:** Experiment I: after introducing the new form f_{pr} , agent switch directly (20-50 simulation steps) to the stable strategy pair $\langle S_6, H_9 \rangle$ (“surprisal” strategy). **Right:** Experiment II: Agents stabilize on $\langle S_{10}, H_5 \rangle$ (categorical-PROG state of the PROG-path) or $\langle S_5, H_{10} \rangle$ (categorical-HAB state of the HAB-path). Final strategy pair $\langle S_{15}, H_3 \rangle$ is never reached. (Gray: unstable states.)

Experiment I is our baseline, not containing any additional assumptions. Its results are depicted in Figure 2 (left). With only one message f_{im} available during the first 1,000 simulation steps, agents expectedly play $\langle S_0, H_3 \rangle$. After the introduction of the second form f_{pr} , all agents switch rapidly, in about 20-50 simulation steps, to the strategy pair $\langle S_6, H_9 \rangle$. This is the “surprisal” strategy: the new form f_{pr} signals that the intended state is context-unusual. Importantly, this strategy was never observed in human languages for the imperfective domain.

Though the emerging strategy pair $\langle S_6, H_9 \rangle$ is linguistically odd, Experiment II shows that we can rule it out by adding a simple and linguistically natural assumption. In real-life interactions, sometimes the hearer would not be able to observe the speaker’s context c . We model that by randomly withdrawing the context cue in 20% of the interactions. This small change has an tremendous effect, Fig. 2, right. Now “surprisal” $\langle S_6, H_9 \rangle$ never emerged, and instead the population stabilized either on strategy pair $\langle S_{10}, H_5 \rangle$ (the categorical-PROG state of the PROG-path), or on $\langle S_5, H_{10} \rangle$ (categorical-HAB state of the HAB-path). The empirical estimate of the probability for each path to emerge was .5.

There are two aspects that differ between the results of Experiment II and what is considered to be empirically observed diachronic trajectories in the imperfective domain Deo (2015). (1) The emergent-PROG $\langle S_2, H_1 \rangle$ and emergent-HAB $\langle S_4, H_{11} \rangle$ states are only short intermezzos in our model, while in reality they can be maintained for several centuries (e.g., both Shakespeare and Laurence Sterne used emergent-PROG). (2) Both PROG and HAB paths in Experiment II do not go towards the single-form state $\langle S_{15}, H_3 \rangle$. This is empirically correct for HAB, but incorrect for PROG. Here, we leave issue (1) aside, and concentrate on issue (2).^d We divide it into two sub-issues: (2a) how to achieve the simplification of

^dWe conjecture that their instability may be caused by the fact that we sometimes withdraw the contextual cue: unlike the categorical systems, which ignore the cue completely, emergent-PROG and emergent-HAB crucially rely on it. Thus it is not surprising that when the cue is withdrawn, these strategies have a hard time. The question is, what other property of the real-life imperfective communication makes those systems relatively stable?

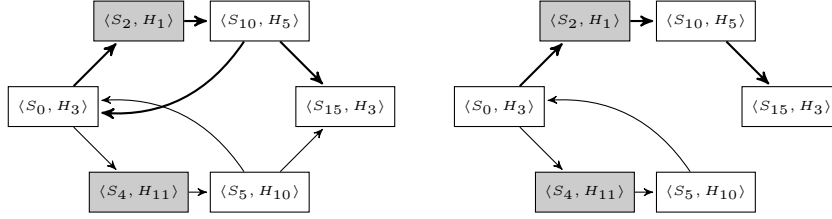


Figure 3. **Left:** Experiment III: The population switches finally to a one-message system, either $\langle S_0, H_3 \rangle$ or $\langle S_{15}, H_3 \rangle$, each equiprobable for both paths. **Right:** Experiment IV: for the HAB-path the population switches back to the initial situation, for the PROG-path the population completes the assumed cycle and switches to the final state $\langle S_{15}, H_3 \rangle$ (gray: unstable states).

a two-form system into a single-form system? (2b) how to derive the asymmetry between the PROG-path and HAB-path?

Consider (2a) first. A two-form system like categorical-PROG or categorical-HAB is perfectly efficient, always achieving communicative success. Why would it then be replaced by a less efficient single-form system? Intuitively, this would also happen if maintaining the efficient two-form system somehow becomes burdensome. Realistically, this could be a result of the “aging” of the old f_{im} , which can be a natural last stage of grammaticalization. If f_{im} becomes less and less suitable for use, there will be an incentive to generalize f_{pr} . Here, we do not test the effects of such a process, studying instead the following modeling alternative: in Experiment III, we gradually increase the cost k of maintaining the two-form system. The effects of this assumption are symmetric for both forms, so we do not hardwire the loss of f_{im} into the model.

Formally, we assign higher cost $c(f)$ to f with lower $fr(f)$, namely the number of interactions an agent encountered f : $c(f) = \alpha \times (1.0 - \frac{fr(f)}{\sum_{f' \in F} fr(f')})$. Note that $\forall f : 0 \leq c(f) \leq \alpha$, so higher α enables higher costs. Note also that a single form is not affected by costs: if $fr(f) = \sum_{f' \in F} fr(f')$ then $c(f) = 0$, no matter how great the α -value is. Thus by increasing the α -value over time, we put two-form systems at increased disadvantage. In Experiment III, we augmented the model of Experiment II with costs defined as above, and increased α by 0.01 after every 1,000 simulation steps. The resulting paths are depicted in Figure 3 (left). Like in Experiment II, the population first stabilizes on categorical-PROG $\langle S_{10}, H_5 \rangle$ or categorical-HAB $\langle S_5, H_{10} \rangle$. But after a while the costs to maintain a two-form system become too high, and the population switches to a one-form system, either $\langle S_0, H_3 \rangle$ (losing f_{pr}) or $\langle S_{15}, H_3 \rangle$ (losing f_{im}). The switch in both directions was equiprobable for both paths.

We turn to (2b). In Experiment III, both PROG and HAB may generalize to a new all-purpose imperfective, and both may be lost in favor of the old IMP form. But we want PROG to generalize, and HAB to never do that. What causes such asymmetry? Deo (2015) conjectures that it might be due to an asymmetry of input

during early language acquisition: “this asymmetry likely stems from the nature of the input to the child, specifically the relative prevalence of PROG forms vs. HAB forms in caregiver speech. [...] this asymmetry in the frequency of phenomenal vs. structural inquiries in child-directed speech would lead to learners generalizing the PROG form rather than any specialized HAB form since exposure to the latter is likely to be less frequent” (Deo (2015, p. 22)). This hypothesis is easy to test in our system: agents of a low age may be presented with state t_s much less often than with t_p . We start with 0 probability of t_s at age 0, and increase it uniformly towards .5 at each step until age C_{max} : $P(t_s | age = a) = \frac{0.5}{\max(C_{max} - a, 1)}$. In Experiment IV, we set $C_{max} = 200$.

This childhood input asymmetry leads to the emergence of exactly the desired trajectories, Fig. 3 (right). From the categorical-HAB state, the system reverts to the initial state after the costs for having two forms rise too high. But if the population enters the PROG-path, then it always generalizes PROG to a new all-purpose imperfective, $\langle S_{15}, H_3 \rangle$, where f_{pr} is the new generalized form. From here on, new emerging forms can realize another cycle of the same shape, and Deo’s conjecture leads to the desired result in our model.

5. Conclusion

We used experiments with reinforcement learning agents playing the Basic Imperfective Game of Deo (2015) with the full strategy space to investigate whether the empirically observed grammar changes involving the imperfective IMP, progressive PROG and habitual HAB would emerge in this setting. With relatively simple assumptions, we achieved the emergence of both the PROG-cycle and the deadlocking HAB-trajectory. The assumptions that we sequentially added to the basic model and their consequences were: (1) the emergence of linguistically odd “surprisal” strategy $\langle S_6, H_9 \rangle$ was prevented by sometimes withdrawing the context cue from the hearer; (2) the switch from a perfectly communicatively efficient two-form grammar calls for a cost function was forced by a gradually increasing symmetric cost for having two forms; and (3) when agents were mostly presented with phenomenal statements in the childhood, the empirically observed PROG-to-IMP cycle emerged, while the unattested hypothetical HAB-to-IMP cycle was ruled out — vindicating a conjecture by Deo (2015).

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